Discussion on: "Optimality Properties and Driver Input Parameterization for Trail-braking Cornering"

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The paper by E. Velenis, P. Tsiotras, and J. Lu deals systematically with how to model the technique of rally driving, where the main concern is the Trail-Braking maneuver. Trail Braking makes it possible to control vehicles at high speed during cornering. The idea in this paper is to find the algorithm or the rules that will result in a time-optimized cornering maneuver, which is used by expert human drivers when competing in off-road car racing. Experimental data were collected from expert rally drivers and then used to extract the driving guidelines and to evaluate the driving-strategy results obtained using optimization on a simulated model of a car. To reduce the computational complexity of the optimization an innovative and useful parameterization of the parameters' search space is suggested. The obtained results, using a simplified car model and the more realistic CarSim simulator, are discussed and validated on the basis of the data collected during the execution of several maneuvers by an expert rally-driving instructor. The goal of the test was to capture the actions of the driver – steering, braking and throttle commands – and the resulting vehicle response, treated as a rigid body. To the best of our knowledge this investigation is a pioneering approach to comparing rally drivers' experience with mathematical models.

In general, the fastest curve through a corner is the one that allows the greatest radius. During a rally the goal is to minimize the overall driving time on the track, which consists of both corners and straight sections. Therefore, the most important objective in any corner is a fast exit speed, which means the car must be straightened up on the exit from the corner as early as possible, so the driver can open the throttle as early as possible [1]. In a series of corners the last exit before the straight is the most important segment. Rally-driving maneuvers, including Trail-Braking, are well known techniques, usually consisting of open-loop recipes. The guidelines for Trail-Braking maneuvers are as follows. The maneuver begins by applying the vehicle's brakes without steering, with the driver adjusting the brake pressure such that the maximum available friction is generated by the tires. This means that the maximum available deceleration is realized, and as a result no friction is available for steering. As the vehicle approaches the corner, the driver starts to steer the vehicle. In Trail-Braking this is done by progressively releasing the brake, in order to allow the cornering forces to act on the tires, and simultaneously – and progressively – increasing the steering angle. As the vehicle decelerates, the weight of the vehicle transfers from the rear axle to the front axle and thus, the front tires generate a higher cornering force than the rear tires. Due to the increased cornering force acting on the front axle and the decreased cornering force acting on the rear axle, the vehicle tends to over steer and rotates about the vertical axis. As the vehicle reaches the apex and its path is aligned with the exit from the corner, the driver accelerates and counter steers to stop the rotation of the vehicle and starts to accelerate toward the exit of the corner. The acceleration causes the weight of the vehicle to transfer from the front axle to the rear axle. As a result, the rear tires generate more friction, so resisting the rotation of the vehicle. The collected experimental data are in accordance with the empirical guidelines that have been discussed. This means that the driver applied a progressively decreasing braking force with increasing command of the steering, followed by counter steering and progressive acceleration. This results in an aggressive cornering maneuver, utilizing high vehicle-slip angles.

The main idea of the paper is to find the time-optimal maneuver that will emulate the Trail-Braking cornering maneuver, which seems to be the fastest strategy for driving though a corner. For optimization purposes a low-order mathematical model of the vehicle was used to overcome the numerical problems. The most important and original idea used here is to parameterize the driver's input commands. This means that certain forms of piece-wise linear input command are assumed and that only the braking points have to be found to optimize the overall cornering time. This means that by using nonlinear programming the time of the braking point and the value

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of the driver's input commands at the braking point have to be found. The braking points are defined as follows: $t_{si}, p_{si}, i = 1,\ldots,4$ for the steering command, $t_{bi}, p_{bi}, i = 1,\ldots,4$ for the braking command and $t_{ai}, p_{ai}, i = 1,2$ for the throttle command. The parameterization of the control inputs is a very useful solution for reducing the numerical complexity during optimization and gives us a solution for a variety of corner geometries with different specific boundary conditions. A very important task in the case of numerical optimization is the proper selection of the cost function. Assuming that the trajectory is known at discrete times $0 = t_0 < t_1 < \ldots < t_N = t_f$ the goal of the optimization is to find $t_{si}, p_{si}, t_{bi}, p_{bi}, t_{ai}, p_{ai}$, that minimize the following cost function:

$$
\begin{aligned}
J = {} & W_t t_f + W_r e_r + W_d e_d(t_f) + W_\psi e_\psi(t_f) \\
& + W_\upsilon e_\upsilon(t_f) + W_y e_y(t_f)
\end{aligned}
$$

where t_f is the final time, $e_r = \sum_{k=1}^{N} e_r(t_k)$ is the cumulative absolute position error from the road limits, $e_d(t_f)$ is the absolute value of the lateral deviation of the vehicle from the inner limit of the road at t_f, $e_\psi(t_f)$ is the final absolute orientation error, $e_\psi(t_f)$ is the final absolute lateral velocity of the vehicle, and $e_y(t_f)$ is the final absolute yaw-rate error of the vehicle. The main problem, which is not solved in this paper, remains the weights W_i $(i = t, r, d, \psi, \upsilon, y)$ that are used for non-dimensionalization and to adjust the relative significance between the terms on the right-hand side of the cost function. The parameterization of the weights is not clarified and it seems that they are chosen in an ad-hoc manner. To reduce the optimization space the values of the input commands at the braking points (p_{si}, p_{bi}, p_{ai}) are also fixed. Nevertheless, this simplified optimization scheme was successful in emulating the Trail-Braking maneuver for all corner geometries.

In order to extend this investigation some theory or directions about how to tune the weighting parameters in the cost function need to be proposed. A comparisons and discussion to other related minimal-time control strategies such as presented in [2–5] would be of interest. Also the applicability of the proposed approach to rally driving and other areas would be interesting. Some possible rally- or drifting-competition applications are as follows: a teaching-assistance tool to improve a rally driver's technique in a realistic car-driving simulator (hardware in the loop), offline or online analyses of a rally driver's performance on the track, an optimization search for the optimal maneuvers for all the corners in a complete circuit for a real rally competition. It would also be very interesting to develop autonomous vehicles that could travel as fast as vehicles with human operators. This means it is of great importance to develop mathematical models and control algorithms that will be able to deal with trajectory generation and trajectory tracking under high-speed and abnormal driving conditions.

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